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ABSTRACT

The usefulness of a latent trait model designed for use with multidimensional test data was investigated in two stages. The first stage consisted of generating simulation data to fit the multidimensional extension of the two-parameter logistic model, applying the model to the data, and comparing the resulting estimates with the known parameters. The unidimensional two-parameter logistic model was also applied to these data. In addition to comparing the estimated parameters with the true parameters, the fit of the unidimensional and multidimensional models to the data were compared. The second stage of the study employed real response data. Items were selected from various subtests of a larger test that had been administered to a large sample in such a way as to simulate shorter tests with varying numbers of subtests. The unidimensional and multidimensional models were applied to these data, and the resulting estimates were used to evaluate the fit of the models to the data. The fit of the two models to the data were then compared and it was determined that the multidimensional model more adequately modeled the real data than did the unidimensional model. (Author/PN)

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A Multidimensional Extension of the Two-Parameter Logistic Latent Trait Model

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Latent trait theory has become an increasingly popular area for research and application in recent years. Areas where latent trait theory has been applied include test scoring (Woodcock, 1974), criterion-referenced measurement (Hambleton, Swaminathan, Cook, Eignor, and Gifford, 1978), test equating (Marco, 1977; Rentz and Bashaw, 1977), tailored testing (McKinley and Reckase, 1980), and mastery testing (Patience and Reckase,1978). While many of these applications have been successful, one unsolved problem is repeatedly encountered. Most latent trait models assume unidimensionality. As a result, applications of these models have been limited to areas in which the tests used measure predominantly one factor. When the assumption of unidimensionality is not met, such as is often the case with achievement tests, most latent trait models are inappropriate. The purpose of this research is to describe and apply a latent trait model that is appropriate for use with tests that measure more than one factor. A model for use with dichotomously scored multidimensional item response data will be described, procedures for estimating the parameters of the model will be presented, and an application of the model to simulated and real test data will be described and evaluated.

The objectives of this research are to determine whether the proposed model more adequately models multidimensional test data than does the undimensional version of the model, and to determine whether the results yielded by the application of the model are consistent with the results of another, more established multivariate data reduction procedure, factor analysis.

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Method

The Model

The model selected for this study, a multidimensional extension of the two-parameter logistic model, is given by

$$P_i(\underline{\theta}_j) = \frac{\exp(c_i + \underline{a}_i\underline{\theta}_j)}{1+\exp(c_i + \underline{a}_i\underline{\theta}_j)} \quad ,$$

where $P_i(\underline{\theta}_j)$ is the probability of a correct response to item i by examinee j, c_i is the difficulty parameter for item i, $\underline{a}_i$ is a vector of item discrimination parameters for item i, θ_j is a vector of ability parameters for examinee j, and

$$\underline{a}_i\underline{\theta}_j = \sum_{k=1}^{m} a_{ik}\theta_{jk} \quad ,$$

where a_{ik} is the discrimination parameter for item i on dimension k, θ_{jk} is the ability parameter for examinee j on dimension k, and m is the number of dimensions modeled.

Estimation Procedures

The procedure used for item parameter estimation for the multidimensional model is a modification of the marginal maximum likelihood procedure proposed by Bock and Aitkin (1981). The procedure was modified to make it appropriate for use with the logistic distribution rather than with the normal distribution. For the unidimensional model, item parameter estimation was performed using the LOGIST estimation program (Wood, Wingersky, and Lord, 1976). This procedure is the most commonly used procedure for estimating the parameters of the 3PL model. It can be used for estimating the parameters of the 2PL model by holding the 'pseudo-guessing' parameter constant at zero.

The ability estimation procedure used for the multidimensional model is a conditional maximum likelihood estimation procedure. It employs an iterative estimation routine based on the Newton-Raphson technique. A complete description of the ability estimation procedure is included in McKinley and Reckase (in press). For the unidimensional model ability estimation was performed using the LOGIST program.

Design

The general design of this study involved two stages. The first stage employed simulation data with known true item and person parameters. The second stage involved the use of real test data, sampled to have specified numbers of subtests in order to control to some degree the factor structure of the tests.

In the first stage of the study response data with one, two, and three dimensions were generated using known parameters. The parameters of the unidimensional and multidimensional forms of the model were estimated for these data, and the resulting sets of estimates were compared to the true parameters and to each other.

In the second stage of the study actual response data for a large test with several subtests were sampled in such a way as to simulate tests having one, two, and three subtests. Although the tests were simulated, the item responses were actual item responses from an administration of the large test. The parameters of the unidimensional and multidimensional forms of the model were estimated, and the resulting estimates were compared with each other.

Datasets

Six datasets were employed in this research, three containing simulated item responses and three containing real item responses. One simulation dataset was generated to have one dimension, a second was generated to have two dimensions, and a third dataset was generated to have three dimensions. The first real dataset was constructed so as to have no subtests, the second had two subtests, and the third had three subtests.

The true item parameters for the simulation datasets were selected in the following way. The c-parameters, or item difficulty parameters, were selected from a table of the standard normal distribution. They were sampled to have a mean of approximately zero and a standard deviation of approximately .5. For the one-dimensional data, the a-parameters, or discrimination parameters, were randomly selected from a uniform distribution having a mean of .70 and a standard deviation of .50. These values were selected as representing typical values obtained from the calibration of real response data using a unidimensional two-parameter logistic model.

For the two-dimensional dataset, the difficulty and discrimination parameters selected for the one-dimensional dataset were used, and a second set of discrimination parameters was added using the same selection procedure as was used for the first set. The same procedure was used to select a third set of discrimination parameters to be used for the three-dimensional dataset. All three simulation datasets included data for 50 items and 1000 examinees.

For the real datasets, item responses were sampled from Form 16 of the Texas Grammar, Spelling, and Punctuation (GSP) test (University of Texas, 1978). For the real dataset having no subtests, response data for the spelling subtest of the GSP test were sampled for 1000 examinees and 30 items. For the two-subtest dataset, data were sampled for 1000 examinees for 15 items from the spelling subtest and 15 items from the grammar subtest of the GSP test. For the three-subtest dataset, response data were sampled for 1000 examinees for 10 items from the spelling subtest, 10 items from the grammar subtest, and 10 items from the punctuation subtest of the GSP test. In all cases the items selected were the items having the highest factor loadings on the factor defined by the subtest on which the items appeared. The factor loadings were from a principal components analysis of phi coefficients computed using the entire test and sample.

Analyses

Simulation Data Analyses The first analysis performed on the simulation data was to compare the item and person parameter estimates obtained for both the unidimensional and the multidimensional models to the known true parameters. The c-parameter estimates were compared to each other and to the true c-parameters using Pearson product moment correlations. The correlations obtained for the two models were compared using Fisher's r to z transformation. For the unidimensional data the a-parameter estimates were compared to each other and to the true a-parameters using the same procedure.

For the multidimensional data there were different numbers of a-parameter estimates for the unidimensional and multidimensional forms of the model. Therefore, there was no one-to-one correspondence between the two sets of estimates. Because of this, correlations between the two sets of estimates would not be meaningful for evaluating the quality of the estimates. However, such correlations might lead to a better understanding of the relationship between the two forms of the model. Therefore, the intercorrelation

matrices for the a-parameter estimates were computed for the multidimensional data.

Another analysis performed on the simulation data was the computation, for each model, of a mean absolute deviation (MAD) statistic. This statistic is given by

$$MAD_i = \sum_{j=1}^{n} |P_{ij} - x_{ij}| \quad ,$$

where P_{ij} is the probability of a correct response to Item i by Examinee j given the item parameter estimates obtained for the model of interest, x_{ij} is the observed response to Item i by Examinee j, MAD_i is the mean absolute deviation statistic for Item i, and n is the number of examinees. This statistic, an indicant of the ability of the models to predict item responses, was computed for all items for both the unidimensional and multidimensional models, and the mean MAD statistics for both models were compared for the simulation data using analysis of variance techniques. In addition, a principal components solution was obtained on phi coefficients computed for each dataset. A varimax rotated factor solution was also obtained and used to facilitate the interpretation of the results of the other analyses.

Real Data Analyses For the real data the true parameters were not known. Therefore, the first analysis performed on the real data was the computation of the MAD statistics. The MAD statistics for the two models were once again compared using analysis of variance techniques. A principal components analysis was also performed for each of the real datasets, and the varimax rotated factor solution was used to facilitate interpretation of the results.

Results

Simulation Data Analyses

True Item Parameters The true item parameters that were used to generate all of the simulation data are shown in Table 1. The c-parameters, or item difficulties, that are shown were used for all three simulation datasets. The first a-parameter column contains the item discrimination parameters used to generate the one-dimensional data. The second a-parameter column contains the item discrimination parameters that, along with the first set of item discrimination parameters, were used to generate the two-

dimensional dataset. The third column of a-parameters were used with the first two sets to generate the three-dimensional dataset.

Inset Table 1 Here

Factor Analyses Table 2 summarizes the results of the factor analyses performed on the simulation datasets that were generated using the item parameters shown in Table 1. For the one-dimensional data the factor loadings that are shown are for the first principal component from a principal components analysis of phi coefficients. For the two- and three-dimensional data the loadings shown are from a varimax rotated principal components solution.

Insert Table 2 Here

For the one-dimensional data the first two eigenvalues from the principal components analysis were 6.54 and 1.34. These data appear to at least approximate the condition of unidimensionality. For the two-dimensional data the first three eigenvalues were 8.07, 4.03, and 1.25. These data clearly do not approximate the condition of unidimensionality. For the three-dimensional data the first four eigenvalues were 9.12, 4.51, 3.81, and 1.03. Again, these data are clearly not unidimensional.

As can be seen from Table 2, there is a strong relationship between the discrimination parameters of the multidimensional 2PL model and the factor loadings from the factor analysis solutions. Likewise, there is a strong relationship between the eigenvalue structures of the data and the dimensionality of the parameter vectors. These data provide strong evidence for the validity of the multidimensional 2PL model as a model for multidimensional item response data. The next issue that must be addressed is whether the parameters of the model can be accurately estimated. This issue was addressed by the simulation data analyses to be reported next.

One-Dimensional Data Table 3 shows the item parameter estimates that were obtained for both models for the one-dimensional simulation data. Note that for the one-dimensional data parameters were estimated for only one dimension for the multidimensional model. As can be seen from the table, the estimates for the unidimensional simulation data were quite similar for the two models, although the mean discrimination parameter estimates were somewhat higher for the multidimensional model than for the unidimensional model. The correlation of the c-parameter estimate with the true c-parameter was .99 for both models. A test for the significance of the difference between these two correlations using Fisher's r to z transformation indicated that they were not significantly different. The correlation of the a-parameter estimates with the true a-parameter was .98 for the unidimensional model and .99 for the multidimensional model. Again, these two correlations were not significantly different. The correlation of the two sets of c-parameter estimates was .99, as was the correlation between the two sets of a-parameter estimates.

Insert Table 3 Here

The great similarity of the estimates obtained for the two models was expected, since in the unidimensional case the two models are essentially the same model. Any differences that were found between the two sets of estimates were probably the result of differences between the two estimation procedures that were used. As indicated by the correlations that were obtained, the differences found between the two sets of estimates were minimal, involving primarily a difference in scale. The variance of the estimates for the unidimensional model was less than the variance of the estimates for the multidimensional model. A simple rescaling of the estimates to place them on the same scale would have eliminated most of the differences found between the two sets of estimates.

The correlation of the estimates of ability with the true abilities was .91 for the unidimensional model, and .92 for the multidimensional model. The difference between these two correlations was not significant. The correlation of the two sets of ability estimates was .99.

Two-Dimensional Data Table 4 shows the item parameter estimates that were obtained for both models for the two-dimensional simulation data. Because of the different number of parameter estimates yielded by the two models, a direct comparison of the correlations between the two sets of parameter estimates would not be meaningful. Nor would a direct comparison of the correlations of the two sets of estimates with the true parameters. However, in order to facilitate the interpretation of the results of the analyses of these data, the intercorrelations for the true and estimated item parameters were computed. The parameter estimates for the multidimensional version of the model were quite strongly correlated with the true parameters. The correlation for the true and estimated c-parameter for the multidimensional model was .99. For both a-parameters the correlation was .98. For the unidimensional model the c-parameter estimate had a correlation of .98 with the true c-parameter, which was not significantly different from the correlation for the multidimensional model. The two sets of c-parameter estimates had a correlation of .99. The unidimensional a-parameter estimates had a correlation of .47 with the first set of true a-parameters and .53 with the second set of true a-parameters. The correlations between the unidimensional a-parameter estimates and the multidimensional a-parameter estimates was .44 for the first set of a-parameter estimates, and .52 for the second set.

Insert Table 4 Here

The correlations between the true ability parameters and the multidimensional estimates were .91 for both dimensions. The unidimensional ability parameter estimates had a correlation of .68 with the first set of true ability parameters and .70 with the first set of estimates for the multidimensional model. The correlation between the unidimensional estimates and the second set of true ability parameters was .67, while a correlation of .73 was obtained for the unidimensional estimates and the second set of ability parameter estimates for the multidimensional model.

Three-Dimensional Data Table 5 shows the item parameter estimates that were obtained for both models for the three-dimensional simulation data. Once again, the estimates for the multidimensional model had high correlations with the true parameters. The c-parameter estimate had a correlation

of .99 with the true c-parameter. The correlation of the first a-parameter estimate with the true first a-parameter was .98, as was the case for the second a-parameter. For the third set of a-parameters the correlation was .99. For the unidimensional version of the model, the correlation between the c-parameter and the estimated c-parameter was .99. The two sets of c-parameter estimates had a correlation of .99. The correlations obtained between the unidimensional a-parameter estimates and the three sets of true a-parameters were .69,-.26, and -.27. The corresponding correlations between the unidimensional a-parameter estimates and the three sets of multidimensional a-parameter estimates were .73, -.20, and -.27.

Insert Table 5 Here

The correlations between the three sets of ability parameter estimates for the multidimensional model and the three sets of true ability parameters were .91, .90, and .90. The correlations obtained between the unidimensional ability parameter estimates and the three sets of true ability parameters were .57, .49, and .45. The corresponding correlations for the multidimensional estimates and the unidimensional estimates were .59, .48, and .48.

Overall Performance on Simulation Data A final analysis that was performed on the simulation data was an analysis of variance performed on all of the MAD statistics combined. Table 6 shows the mean MAD statistics that were computed for both models for the simulation data. The standard deviations for these statistics are also shown. The dimensionality of the data and the model used were independent variables, with model as a repeated measures factor. The analysis of variance performed on these data yielded the results shown in Table 7.

Insert Tables 6 and 7 here

As can be seen, all of the effects were found to be significant. The test for the significance of the dimensionality effect yielded an $F=13.39$, $p<.001$. Analysis

of the cell means indicates that the models yielded lower mean MAD statistics as the dimensionality of the data increased. The test for the significance of the dimensionality by model interaction yielded an $F=154.22$, $p<.001$. A look at the cell means, reported at the bottom of Table 6, reveals that the mean MAD statistics decreased at a much faster rate for the multidimensional model than for the unidimensional model. As the dimensionality of the data increased, then, the advantage gained by use of the multidimensional model increased.

The test for the model effect yielded an $F=1223.04$, $p<.001$, indicating that across the three sets of response data the multidimensional model yielded significantly lower mean MAD statistics. Paired t-tests were performed on these data to compare the mean MAD statistics yielded by the two models for each level of dimensionality. These t-tests yielded a $t=10.64$, $p<.001$ for the unidimensional data, $t=14.36$, $p<.001$ for the two-dimensional data, and $t=46.30$, $p<.001$ for the three-dimensional data. Regardless of the dimensionality of the data, the multidimensional model performed better than the unidimensional model.

Real Data Analyses

Factor Analyses The results of the principal components analysis of phi coefficients for the three real data datasets are summarized in Table 8. The first two eigenvalues from the principal components analysis of the one-factor data are 4.22 and 1.78. The first three eigenvalues from the principal components analysis of the two-factor data are 3.78, 2.27, and 1.24. For the three-factor data the first four eigenvalues are 3.84, 2.72, 1.64, and 1.29.

Insert Table 8 Here

As can be seen from the results of the factor analyses, the one-factor data do at least approximate unidimensionality. The first principal component is not a particularly large one, but it does seem to be dominant, as reflected by the smallness of the second component. The two-factor data do not approximate the condition of unidimensionality. Rather, they seem to have two main components. This is a reasonable reflection of the subtest

structure of these data. The factor loadings shown in Table 8 for the two-factor data give an accurate picture of the subtest structure of the data, with the first fifteen items having higher loadings on the first factor and the last fifteen items having higher loadings on the second factor. The first fifteen items were taken from the spelling test, and the last fifteen were taken from the grammar test.

The three-factor data results are not as clear. The first ten items were from the spelling test, the second ten were from the grammar test, and the last ten were from the punctuation test. From the results of the factor analysis it can be seen that the spelling items loaded on the first factor, and all of the seond ten items except Item 15 loaded on the second factor. However, the last ten items, which were the punctuation items, tended to load on the second factor with the grammar items. This tendency is reflected in the smallness of the third eigenvalue from the principal components analysis. Only items 15, 26 and 30 had high loadings on the third factor. Thus, while the construction of the one- and two-factor tests was successful, less success was achieved in constructing a three-factor test.

One-Factor Data The item parameter estimates that obtained for the one-factor data for both the unidimensional and the multidimensional models are shown in Table 9. The correlation of the two sets of c-parameters estimates was .93, and the correlation of the two sets of a-parameter estimates was .92. The two sets of ability estimates had a correlation of .86.

Insert Table 9 Here

Two-Factor Data Table 10 shows the item parameter estimates that were obtained for the two models for the two-factor real data. The correlation of the two sets of c-parameter estimates was .96. The correlation of the unidimensional a-parameter estimates with the multidimensional a-parameter estimates was .87 for the first dimension and -.40 for the second dimension. The correlation of the unidimensional ability parameter estimates with the multidimensional ability parameter estimates was .53 for the first dimension and .67 for the second dimension.

Insert Table 10 Here

Three-Factor Data Table 11 shows the unidimensional and multidimensional item parameter estimates that were obtained for the three-factor real item response data. The two sets of c-parameter estimates had a correlation of .99. The correlation between the unidimensional a-parameter estimates and the multidimensional a-parameter estimates was .70 for the first dimension , -.38 for the second dimension, and .04 for the third. The unidimensional ability parameter estimates had a correlation of .33 with the first dimension of the multidimensional ability parameter estimates , .53 with the second dimension, and .47 with the third.

Insert Table 11 Here

Overall Performance on Real Data Table 12 shows the means and standard deviations of the MAD statistics that were computed for the real data. Table 13 summarizes a two-way analysis of variance that was performed on the MAD statistics computed for the three sets of real data. The number of subtests (factors) and the model used were the independent variables. Model was treated as a repeated measures variable.

Insert Tables 12 and 13 Here

As can be seen in Table 13, the number of factors effect was not significant. However, the model effect was significant($F=7.73$, $p<.01$), as was the model by number of factors interaction($F=37.25$, $p<.001$). Paired t-tests performed for each level of factor structure yielded a $t=5.10$, $p<.001$ for the one-factor data, $t=3.62$, $p<.01$ for the two-factor data, and $t=5.96$, $p<.001$ for the three-factor data. It can be seen from the cell means shown in Table 12 that the unidimensional model yielded a lower mean MAD

statistic for the one-factor data, while the multidimensional model yielded lower mean MAD statistics for the two- and three-factor data. Overall, then, the multidimensional model outperformed the unidimensional model, although the estimation procedure used for the unidimensional model seemed to perform better on the real data than did the estimation procedure for the multidimensional model, as was reflected in the results of the analyses of the one-factor data. The advantage of using the multidimensional model became evident when two-factor data were analyzed, and the advantage increased as the number of subtests increased.

Discussion

The purpose of this study was to investigate the feasibility of a multidimensional latent trait model. Several research questions were of interest. First, it was necessary to determine whether the parameters of the multidimensional model could be accurately estimated. No model is useful if the parameters of the model cannot be accurately estimated.

A second research question addressed by this study is whether a multidimensional latent trait model more adequately models multidimensional item response data than does a unidimensional model. If it does not, then it is not useful even if the parameters of the model can be estimated.

A third research question investigated by this study is whether the parameters of the model are related to the types of measures obtained by more established multivariate data reduction procedures. If the results of the application of the multidimensional latent trait model are consistent with the results of the application of a procedure such as factor analysis, it would lend considerable credibility to the model as a measurement tool.

This research was divided into two parts: one part based on simulation data, and one part based on real data. The simulation part of the research was designed to determine whether the multidimensional model could actually be used to model multidimensional item response data, whether the model parameters could be successfully estimated, and whether the model would fit multidimensional simulation data more adequately than the unidimensional version of the model would. The real data part of the study was designed to determine whether the multidimensional model would yield

satisfactory results when applied to real data. In both the real and simulation data parts of the study an attempt was made to assess the consistency of the results of the application of the multidimensional model with the results of the application of factor analysis. The results of the simulation part of the study will be discussed first, and then a discussion of the real data part of the study will be presented.

Simulation Data Analyses

Factor Analysis Results The results of the factor analyses of the simulation data indicated that the attempt to generate multidimensional item response data was successful. There was a clear correspondence between the number of dimensions of the model parameters used to generate the data and the dimensionality of the data as indicated by the factor analyses. In addition, there was a clear relationship between the item discrimination parameters and the factor loadings obtained from the principal components analysis of phi coefficients. Thus, not only was the generation of the data successful, but evidence was obtained for the validity of the model as a multidimensional model

One-Dimensional Data In the one-dimensional case the unidimensional and multidimensional models were essentially the same model. The multidimensional model was just a reformulation of the unidimensional model. Therefore, any differences found between the two models in the unidimensional case are probably due to differences in the estimation procedures used for the two models. Even if the two estimation procedures yielded equal quality estimates, some differences might appear in the mean absolute differences for the two models. The ability estimates for the two models were obtained using maximum likelihood estimation techniques. However, the unidimensional item parameter estimates were obtained using maximum likelihood estimation while the item parameter estimates for the multidimensional model were obtained using a marginal maximum likelihood procedure. It is unclear at this point whether there are inherent advantages in using one estimation procedure or the other. Any differences that do occur for one of these reasons will be evident in the results of the analyses of the unidimensional data. Any differences found between the two models for the unidimensional case will serve as a baseline for evaluating the results of the analyses of multidimensional data.

The results of the analyses of the one-dimensional simulation data indicate that the the multidimensional model performed slightly better than the unidimensional model. The correlations of the true and estimated parameters were not significantly different for the two models, but the analyses of the mean absolute differences computed for the two models indicated that the goodness of fit of the multidimensional model to the data was significantly better than the fit for the unidimensional model. Although the parameter estimates were quite similar for the two models, the multidimensional estimation procedure yielded better fit to the data than the unidimensional estimation procedure did. The differences in mean absolute differences for the one-dimensional data serves as a baseline for evaluating the results of the analyses of the two- and three-dimensional data. If there is any advantage to using a multidimensional model, the difference between the mean absolute differences for the two models must be greater for the two- and three-dimensional data than for the unidimensional data.

Two-Dimensional Data The results of the analyses of the two-dimensional simulation data indicate that there is some advantage to using the multidimensional model. The correlations of the estimated and true parameters for the multidimensional model indicate that for two-dimensional simulation data the parameters of the model can be accurately estimated. The mean absolute differences analyses indicate that the multidimensional model yields significantly better goodness of fit to the two-dimensional data than the unidimensional model. It is unclear how much of the difference between the two models is due to differences in the estimation procedures, but the results of the analyses of the unidimensional data indicate that at least part of the difference is due to better estimation for the multidimensional model.

Three-Dimensional Data As was the case for the two-dimensional data, for the three-dimensional data the multidimensional model yielded parameter estimates that were highly correlated with the true parameters. From these results it appears that even with higher dimensionality the parameters of the multidimensional model can be accurately estimated. The mean absolute differences analyses indicate that the multidimensional model yields better fit to the three-dimensional data than the unidimensional model. Again, at least part of the difference between the two models is due to differences in the estimation procedures.

Overall Performance on Simulation Data While there is no apparent advantage to using the c-parameter rather than the

b-parameter in the unidimensional case, it is clear that using the multidimensional model for multidimensional data yields much better fit of the model to the data than could be obtained using the unidimensional model. For the unidimensional case there is very little difference between the two models, but as the dimensionality of the data increases so do the advantages of using the multidimensional model. Of course, these conclusions are based on the analysis of simulation data generated to fit the multidimensional model. Any final conclusions regarding the value of using the multidimensional model must be based not only on the results of simulation data analyses, but also on the results of real data analyses.

Real Data Analyses

Factor Analysis Results The results of the factor analyses performed on the real data indicate that the attempt to construct realistic multidimensional data was successful. The one-factor data had one dominant factor, and the two-factor data had two roughly equal factors. The three-factor data had two large factors and a third smaller factor. Thus, with the exception of the smallness of the third factor of the three-factor data, the factor structure of the real data closely paralleled the subtest structure of the data.

One-Factor Data For the one-factor real data the fit of the unidimensional model to the data was better than the fit of the multidimensional model. The estimation procedure used for the unidimensional model appears to be more robust to violations of the assumptions of the model that are found in real data than is the case for the estimation procedure used for the multidimensional model.

Two-Factor Data The results of the analyses of the two-factor data indicate that the fit of the multidimensional model to these data was significantly better than the fit of the unidimensional model. Thus, the advantages of using a multidimensional model with multidimensional real data are sufficient to overcome any advantage the unidimensional model may have had on the basis of the estimation procedures.

Three-Factor Data The results of the analyses of the three-factor data were consistent with the results of the two-factor data analyses. The fit of the multidimensional model to the three-factor data was better than the fit of the unidimensional model to the data. This is consistent with the results of the simulation data analyses.

Overall Performance on Real Data The analyses of the one-factor data indicate that the estimation procedure used for the unidimensional model may be somewhat better than the procedure used for the multidimensional model when applied to real data. However, whatever disadvantage the multidimensional model may have had due to the estimation procedure was overcome when the models were applied to multidimensional data. As the number of subtests in the real data increased, the difference in the fit of the two models to the data also increased.

Summary and Conclusions

The primary objective of the present research was to investigate the feasibility of a multidimensional latent trait model. The motivation behind this research was a desire to determine whether the great benefits realized through the use of unidimensional latent trait models could also be realized with a multidimensional model. A two-parameter logistic latent trait model and its multidimensional extension were selected for this research.

The design of the study employed two stages. The first stage consisted of generating simulation data to fit the multidimensional extension of the two-parameter logistic model, applying the model to the data, and comparing the resulting estimates with the known parameters. The unidimensional two-parameter logistic model was also applied to these data. In addition to comparing the estimated parameters with the true parameters, the fit of the unidimensional and multidimensional models to the data were compared. The second stage of the study employed real response data. Items were selected from various subtests of a larger test that had been administered to a large sample in such a way as to simulate shorter tests with varying numbers of subtests. The unidimensional and multidimensional models were applied to these data, and the resulting estimates were used to evaluate the fit of the models to the data. The fit of the two models to the data were then compared to determine whether the multidimensional model more adequately modeled the real data than did the unidimensional model.

The results of the analysis of the simulation data indicated that the parameters of the multidimensional model could be accurately estimated. The results of the goodness of fit analyses indicated that the multidimensional model could more adequately model simulated multidimensional response data than did the unidimensional model. The increase in dimensionality of the simulation data did not greatly reduce the accuracy with which the parameters of the multidimensional model could be estimated.

The results of the analysis of the real test data indicated that the multidimensional model also more adequately modeled multidimensional real data than did the unidimensional model. The use of a multidimensional latent trait model does seem to be feasible, and the advantages gained by using such models seem to be great enough to warrant further research into their use.

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TABLE 1
True Item Parameters Used to Generate Simulated Item Response Data

Item	c	a_1	a_2	a_3
1	0.350	1.400	0.300	0.150
2	-0.250	0.300	1.300	0.150
3	-1.150	0.100	0.300	1.650
4	-0.550	1.500	0.200	0.250
5	-0.050	0.350	1.350	0.200
6	1.000	0.150	0.300	1.600
7	-0.400	1.550	0.100	0.250
8	-0.700	0.400	1.700	0.150
9	0.300	0.400	0.250	1.750
10	-0.500	1.650	0.200	0.300
11	-0.100	0.200	1.300	0.150
12	1.050	0.350	0.150	1.600
13	-0.500	1.600	0.200	0.150
14	1.750	0.350	1.450	0.250
15	-1.100	0.200	0.150	1.400
16	0.100	1.750	0.200	0.350
17	-0.200	0.400	1.700	0.250
18	0.550	0.200	0.200	1.550
19	0.400	1.500	0.350	0.350
20	0.250	0.400	1.450	0.250
21	0.650	0.100	0.450	1.500
22	0.100	1.500	0.150	0.250
23	-0.350	0.200	1.600	0.250
24	-0.150	0.300	0.100	1.550
25	0.300	1.350	0.150	0.200
26	0.300	0.350	1.700	0.200
27	-0.300	0.150	0.300	1.750
28	0.400	1.600	0.400	0.250
29	-0.400	0.350	1.700	0.250
30	-0.400	0.350	0.150	1.700
31	1.600	1.450	0.550	0.200
32	-1.000	0.150	1.450	0.100
33	-0.500	0.400	0.250	1.500
34	0.050	1.750	0.300	0.100
35	0.450	0.300	1.600	0.200
36	-0.300	0.300	0.300	1.500
37	-0.900	1.450	0.200	0.300
38	0.400	0.200	1.400	0.300
39	0.250	0.250	0.200	1.500
40	0.150	1.550	0.350	0.400
41	0.350	0.300	1.500	0.150
42	-0.350	0.150	0.300	1.700
43	-0.200	1.350	0.350	0.400
44	0.100	0.250	1.450	0.400
45	0.150	0.150	0.150	1.650
46	-0.150	1.700	0.250	0.200
47	0.350	0.150	1.700	0.100
48	-0.300	0.150	0.400	1.600
49	0.200	1.650	0.150	0.100
50	-0.300	0.400	1.550	0.350
Mean	-0.003	0.703	0.685	0.668
S.D.	0.592	0.622	0.619	0.648

TABLE 2
Factor Loadings Obtained for the One-, Two-, and Three-Dimensional Simulated Item Response Data

Item	One Dimensional	Two Dimensional		Three Dimensional		
	I	I	II	I	II	III
1	0.54	0.60	0.07	0.56	0.07	0.13
2	0.20	0.09	0.57	0.13	0.04	0.52
3	0.07	0.06	0.19	0.05	0.58	0.07
4	0.56	0.54	0.10	0.56	0.08	0.09
5	0.20	0.14	0.53	0.09	0.08	0.56
6	0.07	0.11	0.13	0.05	0.59	0.05
7	0.55	0.60	0.01	0.55	0.08	-0.02
8	0.22	0.11	0.57	0.12	0.04	0.60
9	0.22	0.25	0.12	0.11	0.64	0.03
10	0.62	0.58	0.11	0.61	0.09	0.03
11	0.12	0.10	0.56	0.08	0.08	0.54
12	0.18	0.12	0.05	0.13	0.54	0.08
13	0.55	0.57	0.06	0.55	0.06	0.10
14	0.15	0.06	0.47	0.16	0.04	0.49
15	0.08	0.06	0.04	0.11	0.57	0.08
16	0.62	0.60	0.07	0.58	0.11	0.15
17	0.25	0.14	0.59	0.10	0.10	0.61
18	0.19	0.15	0.13	0.08	0.58	0.04
19	0.57	0.58	0.15	0.58	0.13	0.05
20	0.18	0.17	0.50	0.16	0.01	0.53
21	0.03	-0.01	0.24	0.01	0.56	0.16
22	0.58	0.56	0.07	0.59	0.13	0.04
23	0.19	0.04	0.66	0.12	0.10	0.55
24	0.19	0.18	0.06	0.09	0.62	0.05
25	0.53	0.56	0.07	0.52	-0.02	0.10
26	0.21	0.16	0.60	0.11	0.06	0.61
27	0.12	0.05	0.13	0.01	0.62	0.08
28	0.60	0.55	0.14	0.61	0.02	0.15
29	0.17	0.14	0.62	0.11	0.04	0.60
30	0.23	0.17	0.16	0.14	0.60	0.05
31	0.48	0.50	0.09	0.50	0.07	0.15
32	0.00	0.04	0.54	0.02	0.04	0.49
33	0.24	0.23	0.08	0.11	0.57	0.09
34	0.64	0.63	0.08	0.63	0.04	0.15
35	0.17	0.11	0.62	0.10	0.07	0.56
36	0.17	0.16	0.15	0.12	0.58	0.07
37	0.54	0.56	0.06	0.55	0.12	0.09
38	0.16	0.09	0.58	-0.01	0.09	0.54
39	0.11	0.05	0.06	0.08	0.57	0.05
40	0.56	0.54	0.06	0.60	0.13	0.12
41	0.16	0.14	0.55	0.09	0.10	0.55
42	0.05	0.11	0.17	0.01	0.65	0.10
43	0.62	0.57	0.13	0.55	0.16	0.11
44	0.16	0.09	0.52	0.07	0.12	0.56
45	0.07	-0.03	0.10	0.04	0.61	0.01
46	0.62	0.64	0.03	0.61	0.02	0.09
47	0.11	0.03	0.56	0.07	0.03	0.60
48	0.09	0.07	0.25	0.05	0.62	0.16
49	0.60	0.60	0.01	0.63	0.00	0.14
50	0.19	0.21	0.60	0.09	0.12	0.56

TABLE 3
Item Parameter Estimates for Unidimensional and Multidimensional Models for the One-Dimensional Simulated Item Response Data

Item	Unidimensional		Multidimensional	
	c	a	c	a
1	0.226	0.887	0.323	1.120
2	-0.171	0.227	-0.294	0.309
3	-0.696	0.080	-1.183	0.139
4	-0.233	0.942	-0.409	1.213
5	-0.089	0.230	-0.158	0.323
6	0.523	0.101	0.881	0.113
7	-0.274	0.962	-0.439	1.167
8	-0.401	0.272	-0.679	0.370
9	0.157	0.259	0.255	0.361
10	-0.321	1.291	-0.485	1.417
11	-0.076	0.133	-0.135	0.210
12	0.521	0.210	0.874	0.311
13	-0.282	0.920	-0.475	1.176
14	1.137	0.228	1.912	0.325
15	-0.634	0.084	-1.086	0.152
16	0.100	1.200	0.127	1.363
17	-0.172	0.286	-0.300	0.436
18	0.324	0.221	0.541	0.311
19	0.230	0.991	0.331	1.230
20	0.100	0.206	0.164	0.288
21	0.408	0.041	0.691	0.064
22	0.015	1.011	-0.004	1.231
23	-0.211	0.218	-0.362	0.308
24	-0.119	0.220	-0.207	0.287
25	0.243	0.835	0.377	1.100
26	-0.140	0.210	-0.246	0.337
27	-0.158	0.155	-0.270	0.163
28	0.292	1.126	0.439	1.340
29	-0.219	0.202	-0.378	0.283
30	-0.258	0.271	-0.441	0.375
31	0.937	0.884	1.439	1.100
32	-0.596	0.010	-1.016	0.019
33	-0.293	0.288	-0.501	0.410
34	-0.006	1.315	-0.069	1.461
35	0.268	0.196	0.447	0.284
36	-0.192	0.197	-0.331	0.280
37	-0.470	0.906	-0.771	1.176
38	0.315	0.171	0.528	0.262
39	0.117	0.124	0.196	0.161
40	0.058	0.935	0.075	1.177
41	0.219	0.164	0.366	0.260
42	-0.162	0.084	-0.275	0.093
43	-0.179	1.228	-0.285	1.395
44	0.034	0.181	0.051	0.237
45	0.068	0.085	0.115	0.110
46	-0.114	1.208	-0.221	1.382
47	0.186	0.108	0.315	0.201
48	-0.151	0.105	-0.263	0.131
49	0.146	1.072	0.219	1.327
50	-0.157	0.204	-0.273	0.297
Mean	0.003	0.470	-0.018	0.592
S.D.	0.351	0.428	0.581	0.495

TABLE 4
Item Parameter Estimates for Unidimensional and Multidimensional Models for the Two-Dimensional Simulated Item Response Data

Item	Unidimensional		Multidimensional		
	c	a	c	a_1	a_2
1	0.182	0.707	0.246	1.322	0.128
2	-0.119	0.652	-0.306	0.085	1.260
3	-0.617	0.202	-1.075	0.118	0.292
4	-0.193	0.678	-0.455	1.121	0.273
5	0.087	0.594	0.090	0.198	1.137
6	0.685	0.229	1.140	0.172	0.229
7	-0.073	0.599	-0.250	1.265	0.066
8	-0.350	0.737	-0.742	0.131	1.300
9	0.213	0.310	0.315	0.422	0.182
10	-0.139	0.741	-0.384	1.234	0.206
11	-0.054	0.691	-0.170	0.159	1.155
12	0.598	0.148	1.009	0.207	0.100
13	-0.200	0.657	-0.508	1.213	0.133
14	0.905	0.577	1.609	0.086	1.098
15	-0.552	0.068	-0.949	0.127	0.105
16	0.092	0.738	0.042	1.306	0.185
17	-0.069	0.822	-0.203	0.185	1.268
18	0.296	0.226	0.474	0.239	0.206
19	0.384	0.863	0.548	1.304	0.339
20	0.221	0.700	0.324	0.305	1.081
21	0.377	0.198	0.657	-0.038	0.367
22	0.046	0.640	-0.033	1.174	0.139
23	-0.084	0.774	-0.238	-0.014	1.494
24	-0.113	0.180	-0.225	0.282	0.107
25	0.207	0.657	0.283	1.193	0.158
26	-0.179	0.894	-0.406	0.338	1.356
27	-0.142	0.159	-0.253	0.097	0.171
28	0.358	0.785	0.506	1.203	0.321
29	-0.203	0.898	-0.453	0.258	1.421
30	-0.198	0.275	-0.354	0.287	0.252
31	0.906	0.690	1.568	1.284	0.129
32	-0.532	0.599	-1.093	-0.053	1.249
33	-0.283	0.254	-0.523	0.374	0.127
34	0.073	0.799	0.030	1.391	0.210
35	0.232	0.819	0.381	0.226	1.377
36	-0.198	0.239	-0.364	0.252	0.247
37	-0.380	0.649	-0.838	1.228	0.116
38	0.345	0.727	0.602	0.129	1.274
39	0.193	0.088	0.322	0.077	0.083
40	0.212	0.641	0.280	1.134	0.118
41	0.187	0.755	0.279	0.215	1.194
42	-0.212	0.221	-0.394	0.160	0.228
43	0.016	0.775	-0.096	1.203	0.282
44	0.090	0.579	0.100	0.099	1.050
45	0.060	0.041	0.098	0.027	0.144
46	-0.012	0.720	-0.185	1.433	0.063
47	0.241	0.572	0.449	-0.039	1.199
48	-0.215	0.262	-0.379	0.102	0.396
49	0.156	0.613	0.181	1.274	0.032
50	-0.118	1.015	-0.289	0.412	1.382
Mean	-0.043	0.551	0.007	0.537	0.549
S.D.	0.326	0.265	0.585	0.531	0.521

TABLE 5
Item Parameter Estimates for Unidimensional and Multidimensional Models for the Three-Dimensional Simulated Item Response Data

Item	Unidimensional		Multidimensional			
	c	a	c	a_1	a_2	a_3
1	0.167	0.703	0.282	1.174	0.300	0.152
2	-0.117	0.530	-0.241	0.250	0.988	0.142
3	-0.536	0.553	-1.214	0.173	0.171	1.233
4	-0.294	0.661	-0.630	1.122	0.223	0.170
5	-0.014	0.562	-0.079	0.146	1.070	0.188
6	0.557	0.542	1.371	0.171	0.326	1.521
7	-0.189	0.515	-0.406	1.075	-0.030	0.106
8	-0.261	0.628	-0.606	0.269	1.348	0.213
9	0.173	0.609	0.347	0.232	0.041	1.340
10	-0.258	0.674	-0.621	1.355	-0.041	0.204
11	-0.098	0.535	-0.215	0.133	1.036	0.217
12	0.600	0.624	1.182	0.306	0.071	1.180
13	-0.331	0.647	-0.681	1.084	0.173	0.132
14	0.930	0.659	1.956	0.583	1.314	0.049
15	-0.538	0.630	-1.162	0.325	0.099	1.240
16	0.165	0.816	0.365	1.272	0.278	0.265
17	-0.104	0.672	-0.349	0.213	1.339	0.265
18	0.394	0.549	0.845	0.062	0.153	1.255
19	0.278	0.705	0.512	1.233	0.076	0.295
20	0.066	0.550	0.124	0.373	0.996	0.092
21	0.319	0.559	0.673	0.033	0.333	1.190
22	0.051	0.698	0.033	1.224	0.050	0.326
23	-0.190	0.626	-0.356	0.272	1.117	0.267
24	0.033	0.581	0.052	0.157	0.042	1.258
25	0.141	0.503	0.196	0.957	0.200	-0.025
26	-0.210	0.655	-0.491	0.226	1.405	0.227
27	-0.136	0.522	-0.306	0.052	0.150	1.271
28	0.216	0.757	0.433	1.324	0.306	-0.002
29	-0.135	0.604	-0.360	0.259	1.238	0.208
30	-0.184	0.633	-0.443	0.269	0.093	1.270
31	0.917	0.774	1.727	1.212	0.381	0.003
32	-0.458	0.405	-0.927	0.013	0.967	0.051
33	-0.223	0.623	-0.477	0.324	0.241	1.136
34	0.014	0.802	-0.021	1.424	0.301	0.056
35	0.225	0.576	0.457	0.201	1.187	0.206
36	-0.135	0.597	-0.375	0.282	0.142	1.207
37	-0.480	0.704	-1.044	1.230	0.191	0.311
38	0.198	0.462	0.263	-0.055	1.034	0.190
39	0.165	0.521	0.355	0.224	0.039	1.186
40	0.069	0.840	0.027	1.268	0.262	0.307
41	0.140	0.563	0.259	0.189	1.068	0.198
42	-0.139	0.578	-0.252	0.081	0.208	1.465
43	-0.121	0.773	-0.301	1.100	0.229	0.347
44	0.033	0.594	-0.007	0.134	1.122	0.357
45	0.092	0.462	0.166	0.066	-0.012	1.268
46	-0.114	0.646	-0.300	1.295	0.100	0.077
47	0.209	0.536	0.406	0.063	1.199	0.122
48	-0.085	0.652	-0.224	0.004	0.380	1.405
49	0.190	0.722	0.308	1.392	0.250	-0.009
50	-0.171	0.615	-0.321	0.118	1.148	0.291
Mean	-0.164	0.619	0.001	0.538	0.506	0.528
S.D.	0.313	0.094	0.659	0.510	0.484	0.529

TABLE 6
Descriptive Statistics for MAD Statistics Obtained for the Simulation Datasets

No. of Dimensions	Statistic	Model	
		Unidimensional	Multidimensional
1	Mean	0.434	0.407
	S.D.	0.059	0.076
2	Mean	0.428	0.356
	S.D.	0.035	0.067
3	Mean	0.427	0.309
	S.D.	0.024	0.028

TABLE 7
Two-Way Analysis of Variance on Mean Absolute Differences with Dimensionality of Data and Model as Independent Measures with Repeated Measures over Model

Source	SS	df	MS	F	p
Dimensionality	0.136	2	0.068	13.390	0.000
Error	0.749	147	0.005		
Model	0.390	1	0.390	1223.040	0.000
Model x Dim.	0.098	2	0.049	154.220	0.000
Error	0.047	147	0.000		

TABLE 8
Factor Loadings Obtained for the One-, Two-, and Three-Factor Real Item Response Data

Item	One Factor	Two Factor		Three Factor		
	I	I	II	I	II	III
1	0.16	0.57	-0.03	0.56	0.03	-0.07
2	0.15	0.59	0.04	0.60	0.08	0.01
3	0.16	0.37	0.12	0.45	0.17	0.05
4	0.36	0.46	0.16	0.40	0.06	0.00
5	0.25	0.33	0.10	0.58	0.05	0.06
6	0.13	0.36	0.03	0.48	0.05	0.00
7	0.20	0.42	0.02	0.52	0.10	0.07
8	0.24	0.56	0.02	0.48	-0.02	-0.01
9	0.28	0.32	0.08	0.66	-0.02	0.06
10	0.25	0.48	0.01	0.68	-0.02	0.03
11	0.52	0.53	0.08	0.07	0.46	0.05
12	0.32	0.47	-0.06	0.01	0.36	0.07
13	0.29	0.26	0.22	-0.10	0.30	0.10
14	0.55	0.61	-0.00	0.00	0.34	-0.03
15	0.39	0.64	-0.03	-0.01	0.14	0.80
16	0.34	0.11	0.47	0.11	0.21	0.26
17	0.22	0.03	0.38	0.03	0.52	0.14
18	0.49	0.06	0.25	0.07	0.36	0.09
19	0.36	0.01	0.28	-0.03	0.50	0.05
20	0.37	0.09	0.37	0.07	0.35	0.05
21	0.40	0.02	0.35	0.19	0.43	0.00
22	0.51	0.00	0.43	0.06	0.42	0.11
23	0.35	0.12	0.34	0.07	0.28	0.05
24	0.33	0.06	0.61	-0.07	0.43	0.10
25	0.46	0.15	0.36	0.04	0.46	-0.04
26	0.50	0.09	0.24	-0.00	0.10	0.78
27	0.40	0.07	0.46	0.15	0.39	0.19
28	0.34	0.05	0.27	0.08	0.23	0.29
29	0.54	0.02	0.49	0.04	0.44	0.01
30	0.57	0.10	0.33	-0.01	0.08	0.83

TABLE 9
Item Parameter Estimates for Unidimensional and Multidimensional Models for the One-Factor Real Item Response Data

Item	Unidimensional		Multidimensional	
	c	a	c	a
1	1.71	0.38	2.14	0.68
2	0.12	0.26	-0.04	0.22
3	0.31	0.28	0.24	0.25
4	2.10	0.87	1.86	1.03
5	0.58	0.55	0.42	0.29
6	0.51	0.23	0.56	0.30
7	-0.10	0.37	-0.55	0.38
8	0.23	0.51	-0.03	0.24
9	2.09	0.69	2.16	0.91
10	0.69	0.54	0.49	0.44
11	3.49	1.40	2.71	1.76
12	1.03	0.56	0.77	0.79
13	0.08	0.47	-0.59	0.74
14	2.14	1.26	1.37	2.04
15	1.04	0.70	0.46	1.01
16	0.54	0.59	-0.12	0.90
17	0.02	0.30	-0.46	0.51
18	1.03	1.09	0.16	1.49
19	1.16	0.69	0.69	1.01
20	1.46	0.71	1.12	1.01
21	1.66	0.70	1.28	1.24
22	2.24	1.14	1.43	1.41
23	0.72	0.56	0.09	1.01
24	1.28	0.67	1.02	0.80
25	2.30	1.07	1.69	1.35
26	1.40	0.97	0.48	1.32
27	2.78	0.90	2.80	1.42
28	-0.08	0.66	-1.08	0.94
29	2.70	1.26	1.83	1.65
30	2.66	1.45	1.44	1.69
Mean	1.26	0.73	0.81	0.96
S.D.	0.98	0.34	0.99	0.50

TABLE 10
Item Parameter Estimates for Unidimensional and Multidimensional Models for the Two-Factor Real Item Response Data

Item	Unidimensional		Multidimensional		
	c	a	c	a_1	a_2
1	3.49	1.42	3.17	1.42	0.12
2	2.05	1.16	1.45	1.36	0.20
3	0.99	0.59	0.66	0.79	0.28
4	0.89	0.83	0.22	0.93	0.20
5	1.08	0.54	0.97	0.66	0.17
6	1.36	0.56	1.40	0.78	0.16
7	1.60	0.63	1.59	0.97	0.21
8	2.17	1.08	1.80	1.26	0.17
9	0.67	0.42	0.43	0.65	0.17
10	2.13	0.91	2.05	1.00	0.16
11	1.37	0.93	0.80	1.11	0.14
12	2.78	0.92	3.22	1.13	-0.08
13	-0.07	0.48	-0.94	0.69	0.41
14	2.72	1.28	2.27	1.39	0.23
15	2.32	1.13	2.04	1.57	0.01
16	1.15	0.79	1.12	0.47	0.89
17	-1.20	0.50	-2.68	0.33	0.85
18	0.22	0.31	-0.06	0.20	0.41
19	1.09	0.32	1.44	0.02	0.57
20	0.32	0.28	0.33	-0.22	0.51
21	1.35	0.49	1.58	0.10	0.73
22	0.18	0.42	-0.30	0.09	0.78
23	0.76	0.52	0.50	0.31	0.55
24	-0.07	0.87	-0.85	0.26	1.28
25	-0.05	0.18	-0.30	-0.26	0.61
26	0.96	0.43	1.04	0.22	0.32
27	-0.02	0.53	-0.79	0.25	0.73
28	0.53	0.30	0.40	0.20	0.47
29	-0.65	0.60	-1.66	0.07	0.80
30	1.13	0.54	1.11	0.26	0.55
Mean	1.04	0.67	0.73	0.60	0.42
S.D.	1.06	0.32	1.32	0.52	0.31

TABLE 11
Item Parameter Estimates for Unidimensional and Multidimensional Models for the Three-Factor Real Item Response Data

Item	Unidimensional		Multidimensional			
	c	a	c	a_1	a_2	a_3
1	3.08	1.21	3.28	1.13	0.42	-0.09
2	1.78	0.88	1.60	1.18	0.23	-0.05
3	0.78	0.59	0.62	0.99	0.32	0.01
4	1.48	0.48	1.71	0.80	0.30	0.07
5	2.02	0.95	1.87	1.11	0.25	0.11
6	1.99	0.79	2.16	0.87	0.21	0.02
7	1.22	0.71	0.92	0.96	0.16	0.09
8	2.61	0.81	3.27	1.03	0.09	-0.06
9	2.51	1.14	2.44	1.33	0.20	0.22
10	2.05	0.89	2.16	1.42	0.12	0.13
11	1.11	0.75	1.07	0.34	0.95	0.24
12	-1.21	0.41	-2.65	0.11	0.66	0.35
13	0.31	0.28	0.27	-0.21	0.38	0.29
14	1.32	0.46	1.60	0.06	0.63	0.08
15	0.19	0.76	-0.70	0.01	0.51	2.00
16	0.75	0.53	0.49	0.29	0.40	0.34
17	-0.10	0.69	-1.03	0.14	0.79	0.32
18	-0.05	0.40	-0.68	0.19	0.48	0.20
19	-0.67	0.53	-1.64	-0.13	0.74	0.16
20	1.13	0.57	1.06	0.19	0.64	0.10
21	0.14	0.59	-0.62	0.46	0.61	0.06
22	0.29	0.53	-0.08	0.21	0.80	0.27
23	1.63	0.63	1.78	0.26	0.56	0.22
24	0.01	0.41	-0.45	-0.15	0.60	0.18
25	1.38	0.68	1.38	0.10	0.83	0.02
26	0.40	0.70	-0.03	-0.02	0.33	1.66
27	0.73	0.77	0.42	0.44	0.69	0.33
28	0.67	0.53	0.41	0.14	0.34	0.42
29	0.86	0.56	0.61	0.17	0.79	0.17
30	0.63	0.80	0.46	-0.02	0.29	2.05
Mean	0.97	0.67	0.72	0.45	0.48	0.33
S.D.	0.98	0.21	1.37	0.49	0.24	0.55

TABLE 12
Descriptive Statistics for MAD Statistics Obtained for the Real Datasets

No. of Factors	Statistic	Model	
		Unidimensional	Multidimensional
1	Mean	0.300	0.339
	S.D.	0.135	0.117
2	Mean	0.310	0.287
	S.D.	0.123	0.108
3	Mean	0.312	0.285
	S.D.	0.115	0.110

TABLE 13
Two-Way Analysis of Variance on Mean Absolute Differences with Number of Factors and Model as Independent Measures with Repeated Measures over Model

Source	SS	df	MS	F	p
No. of Factors	0.036	2	0.018	0.670	0.516
Error	2.355	87	0.027		
Model	0.007	1	0.007	7.730	0.007
Model x Factors	0.066	2	0.033	37.250	0.000
Error	0.077	87	0.001		